Dynamics of Large-scale Atmospheric Flow

701-1221-00L, HS 22

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1 Equations

1.1 Fundamental equations

1.1.1 Navier-Stokes

$$\frac{D\mathbf{u}}{Dt} = \underbrace{-\frac{1}{\rho}\nabla p}_{\text{Pressure}} \underbrace{-(2\Omega \times \mathbf{u})}_{\text{Coriolis}} \underbrace{- g'K}_{\text{Gravity}} + \underbrace{F^{**}}_{\text{Viscous}} \tag{1}$$

1.1.2 Conservation of mass

$$\frac{D\rho}{Dt} + \rho(\nabla \mathbf{u}) = 0 \tag{2}$$

1.1.3 First law of thermodynamics

$$\frac{D\theta}{Dt} = \left(\frac{\theta}{c_p T}\right)\mathcal{H} \tag{3}$$

if $\mathcal{H} = 0$, the process is *adiabatic*

1.1.4 Equation of state

$$p = \rho R T \tag{4}$$

1.2 Circulation

$$C = \oint_c \vec{v}\, dc = \oint (u\, dx + v\, dy + w\, dz) = \oint_0^{2\pi} \vec{v}\, r\, d\phi \tag{5}$$

1.3 Quasi geostrophic system of equations

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{6}$$

Vorticity equation

$$\frac{D_h}{Dt}\zeta + \beta v = -f_0(\nabla_h \vec{v}) \tag{7}$$

1.4 Geostrophic streamfunction

$$\nabla_h \psi = \zeta_G \tag{8}$$

1.5 Wave theory

Pertubation tendency

$$\psi = \underbrace{\bar{\psi}(y,z)}_{\text{Mean meridional flow}} + \underbrace{\psi'(x,y,z,t)}_{\text{Pertubation}} \tag{9}$$

assuming $|\psi'| \ll |\bar{\psi}|$

$$\left(\frac{\partial}{\partial t} + \mathcal{U}\right)q' + v'\left(\frac{\partial}{\partial y}\bar{q} + \beta\right) = 0 \tag{10}$$

with $\mathcal{U} = -\frac{\partial \bar{\psi}}{\partial y}$

2 Concepts

2.1 Thermal wind

Thermal wind describes the vertical change of geostrophic (i.e. horizontal) wind

$$\frac{\partial}{\partial z}\mathbf{v_G} = \left(\frac{1}{f}\frac{g}{\theta_0}\right)(\mathbf{k} \times \nabla_h \theta^*) \tag{11}$$

2.2 Q-Vector

The Q-Vector indicates if there is cyclogenesis ($\mathcal{F} < 0$, $\mathcal{F} \sim \nabla_h Q$)

How to determine the Q-Vector on weather charts:

1. Locate regions with:
   - Large temperature gradient
   - Strong wind change
2. Determine wind-change vector along η (warm to the right, see Figure 1)
3. Rotate that vector by $-90°$

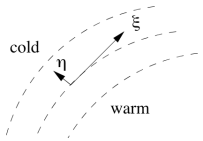

Figure 1: Direction of η

$$\mathbf{Q} = -\frac{g}{\theta_0}|\nabla_h \theta^*|(\mathbf{k} \wedge \frac{\partial}{\partial \xi}\mathbf{v_G}) \tag{12}$$

2.3 PV streamer

- is an upper level positive PV anomaly
- induces cyclonal flow

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References

- Script, Heini Wernli and Lukas Papritz, 2022

Image sources

- Figure 1 (Wernli and Papritz 2022)